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THE SCALAR ETHER-THEORY OF GRAVITATION AND ITS FIRST TEST IN CELESTIAL MECHANICS

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The motivations for investigating a theory of gravitation based on a concept of “ether” are discussed— a crucial point is the existence of an alternative interpretation of special relativity, named the Lorentz-Poincaré ether theory. The basic equations of one such theory of gravity, based on just one scalar field, are presented. To check this theory in celestial mechanics, an “asymptotic” scheme of post-Newtonian (PN) approximation is summarized and its difference with the standard PN scheme is emphasized. The derivation of PN equations of motion for the mass centers, based on the asymptotic scheme, is outlined. They are implemented for the major bodies of the solar system and the prediction for Mercury is compared with an ephemeris based on general relativity.

Keywords: Alternative theories; preferred frame; post-Newtonian; celestial mechanics.

1. Introduction: Why an Ether Theory of Gravitation?

(i) Our first motivation was to extend the *Lorentz-Poincaré ether theory* so that gravitation be included. The Lorentz-Poincaré ether theory may be described as the theory according to which a) the ether is an inertial frame E such that Maxwell’s equations are valid in E , and b) any material object that moves with respect to E undergoes a Lorentz contraction. As shown, in particular, by Prokhovnik,¹ this theory is physically equivalent to standard *special* relativity (SR). This makes its ether undetectable, hence “superfluous” (Einstein 1905). But since SR does not involve gravitation, we may ask whether a preferred frame could exist but remain hidden in the absence of gravitation, and become detectable in its presence.

(ii) An “ether” could help to make quantum theory and gravitation theory compatible. Quantum theory was originally built in a flat space-time, moreover it uses a preferred time. In flat space-time, this is the inertial time, which depends on the inertial frame in a way that remains compatible with Lorentz invariance. Yet in the curved space-time of gravitation, there seems to be no way to prefer some time coordinate, except if we *a priori* admit a “preferred space-time foliation”, i.e. an ether. This may be illustrated already for the case of the Klein-Gordon equation,

for which many possible extensions to curved space-time *a priori* exist, but one is preferred if we have an ether.² Further, quantum theory shows that “vacuum” has physical effects, e.g. the Casimir effect, now experimentally confirmed.

(iii) Investigating a strongly alternative theory also opens a new way to solve some problems common to general relativity (GR) and to most extensions of it: a) Singularities: The investigated “scalar ether-theory” does avoid singularities, in gravitational collapse³ and in cosmology as well.⁴ b) Gauge condition: The solutions to the underdeterminacy of the Einstein equations as a system of partial differential equations are either to say that the Lorentz manifold is determined modulo diffeomorphisms, or to add a gauge condition in a fixed space-time manifold (the latter way is used in applications). What is the precise link between these two solutions? In the scalar ether-theory, there is no need for any gauge, yet space-time is fixed. c) Galactical dark matter: Identified candidates seem poorly found. In the present ether-theory, the preferred-frame effects are probably more important at the galactical scale and beyond, due to the large time scales involved.

2. Basic Equations of the Investigated Theory

N.B.: Most equations are preferred-frame ones with space covariance only.

(i) Gravitation is seen as Archimedes’ thrust in an imagined perfect fluid (“ether”) with pressure p_e and density $\rho_e = \rho_e(p_e)$. This leads to define the *gravity acceleration vector* as follows:³

$$\mathbf{g} = -\frac{\text{grad } p_e}{\rho_e}. \quad (1)$$

Note that, due to Eq. (1), p_e and ρ_e decrease towards the attraction, thus $\rho_e(\mathbf{x}, t) < \rho_e^\infty(t) \equiv \text{Sup}_{\mathbf{x} \in M} \rho_e(\mathbf{x}, t)$ in a gravitational field, where M is the “space” manifold, i.e. the set of the positions \mathbf{x} in the preferred frame E .

(ii) *Assumed metric effects of a gravitational field.*³ We assume that the space-time $\mathbf{R} \times M$ is equipped with a flat metric γ^0 for which the preferred frame E is an inertial (Galilean) frame. The inertial time t in E is called the “absolute time”, and the Euclidean space metric associated with γ^0 in the frame E is denoted by g^0 . Yet we also assume that, in a gravitational field, i.e. $\rho_e(\mathbf{x}, t) < \rho_e^\infty(t)$, there are metric effects, similar to those due to uniform motion (see Ref. 1): the meters are contracted and the clocks are slowed down, in the ratio $\beta \equiv \rho_e(\mathbf{x}, t)/\rho_e^\infty(t)$ and (for the meters) in the direction \mathbf{g} only. This means a dilation (contraction) of the length (time) intervals, when they are indeed measured with physical instruments, as compared with those that would be evaluated in terms of the flat space metric g^0 and the absolute time t . Hence, the “physical” space metric g in the frame E is a Riemannian one, and the measured time is a “local” one, denoted by $t_{\mathbf{x}}$ (at point $\mathbf{x} \in M$). Thus, the “physical” space-time metric γ is a curved Lorentzian metric.

Moreover, SR leads to assuming the relation $p_e = c^2 \rho_e$.

(iii) *Gravitational field equation.* The following equation is stated for the scalar gravitational field p_e :

$$\Delta_g p_e - \frac{1}{c^2} \frac{\partial^2 p_e}{\partial t_{\mathbf{x}}^2} = 4\pi G \sigma \rho_e, \quad (2)$$

where σ is the T^{00} component of the energy-momentum tensor of matter and non-gravitational fields \mathbf{T} , when the time coordinate is $x^0 = ct$ with t the absolute time, and in any spatial coordinates that are adapted to the preferred reference frame E.⁵ The derivative with respect to the local time is defined by

$$\frac{\partial}{\partial t_{\mathbf{x}}} = \frac{1}{\beta(t, \mathbf{x})} \frac{\partial}{\partial t}. \quad (3)$$

In Eq. (2), the Laplace(-Beltrami) operator is defined with the curved space metric g (relative to the frame E). The same is true for the grad operator in Eq. (1).

(iv) *Dynamics is governed by Newton's second law:* force = time-derivative of momentum.⁵ The force over the test particle is the gravitational force $m(v)\mathbf{g}$, plus the nongravitational (e.g. electromagnetic) force, where $m(v)$ is the relativistic inertial mass, involving the Lorentz factor. The momentum is $m(v)\mathbf{v}$; \mathbf{v} and $v = |\mathbf{v}|$ are evaluated with the physical metric. The time-derivative of momentum is uniquely defined from compelling requirements (including Leibniz' rule for a scalar product).⁵ In the *static* case, that extension of Newton's 2nd law *implies Einstein's geodesic motion*. For a dust, we may apply this extension pointwise in the continuum, and it implies a new equation for continuum dynamics:⁶

$$T_{\mu;\nu}^{\nu} = b_{\mu}, \quad b_0 \equiv \frac{1}{2} g_{jk,0} T^{jk}, \quad b_i \equiv -\frac{1}{2} g_{ik,0} T^{0k}. \quad (4)$$

The universality of gravity is expressed in the fact that Eq. (4) is assumed to hold true for any material medium (thus also for a nongravitational field).

3. Asymptotic Post-Newtonian Approximation

An “asymptotic” post-Newtonian approximation (PNA) was developed⁷ (cf. Futamase & Schutz⁸ and Rendall⁹ in GR; in GR, the local field equations of the asymptotic method have not been used to get equations of motion for the mass centers of extended bodies. This has been done in the present theory^{10,11}):

(i) The gravitational field *and* the matter fields are expanded (in the standard PNA,^{12,13} only the gravitational field is expanded).

(ii) For definiteness, each body is assumed to be made of a barotropic perfect fluid (one fluid per body). Other constitutive laws may also be considered, of course.

(iii) A family (S_λ) of gravitating systems is deduced from the given system S . To do this, we use the fact that an exact similarity transformation exists in Newtonian gravity.^{7,8} This transformation is applied to the initial data for S . For this, $V \equiv c^2(1 - f)/2$ is substituted for the Newtonian potential, where $f \equiv \gamma_{00}$ ($1 - f \ll 1$ for a weak field). The initial data for S (the system of interest, e.g. the solar system) is general,⁷ in contrast with Ref. 8 in which the initial space metric was very special. (In Ref. 9, the family was *a priori* assumed.)

Adopting units $[T]_\lambda = [T]/\lambda^{1/2}$ and $[M]_\lambda = \lambda[M]$ for system S_λ , all fields are $\text{ord}(\lambda^0)$, and the small parameter λ is proportional to $1/c^2$ (in fact $\lambda = (c_0/c)^2$, where c_0 is the velocity of light in the starting units $[T]$ and $[M]$). Therefore, the derivation of asymptotic expansions is straightforward.⁷ (In the standard PN scheme, $1/c^2$ is *formally* considered as a small parameter.) That $1/c^2$, not $1/c$, turns out to be the effective small parameter, is due to the fact that it is only $1/c^2$ that enters in the equations. The theory admits consistent expansions in powers of λ (or $1/c^2$). The first (zero-order) term is Newtonian gravity, hence the theory admits a correct Newtonian limit. The first-order approximation in λ or $1/c^2$ is the first PN approximation. Using these expansions is justified insofar as the system of interest corresponds to a small value λ_0 of λ , which is the case, e.g., for the solar system.

4. PN Equations of Motion for the Mass Centers

The mass centers (MC) are defined¹⁰ as local barycenters of the rest-mass density ρ_{exact} (instead of, e.g., the active energy density σ , Eq. (2)), because (i) $\rho_{\text{exact}} = 0$ outside the bodies (which is wrong if one takes instead a density that involves gravitational energy, since the latter is distributed in the whole space) and (ii) ρ_{exact} , or rather its PN approximation, obeys the usual continuity equation, i.e. without adding gravitational energy and its flux. To get the MC's equation of motion, one just integrates the local equations of motion inside the different bodies.^{10,11} Due to the use of the asymptotic method, the local equations of orders 0 and 1 in λ or $1/c^2$ are separated. E.g.:

$$\partial_t \rho_0 + \partial_j (\rho_0 u_0^j) = 0, \quad \partial_t \rho_1 + \partial_j (\rho_1 u_0^j + \rho_0 u_1^j) = 0 \quad (5)$$

for the continuity equation, derived from the time component at the first PNA. The equation of order 1 is linear with respect to the fields of order 1. Therefore, separate equations are also obtained for the MC's, and the equation for PN corrections (order 1) is linear with respect to order-1 quantities. To get tractable equations, every field is decomposed into a self-field and an external field, and account is taken of the "good separation" between different bodies, which means that

$$\eta \equiv \text{Sup}_{a \neq b} (r_a / |\mathbf{x}_a - \mathbf{x}_b|) \ll 1 \quad (6)$$

(r_a is the radius of body (a)). In the solar system, terms up to and including η^3 must be retained.¹¹ A rigid motion, possibly including self-rotation, is assumed for

each body. Finally, the rest-mass density of the order 0, ρ_0 , is assumed spherical for each body, at the stage of calculating the PN *corrections*. One thus gets explicit equations of motion for the mass centers.¹¹ They show that the self-rotation of the bodies *and their internal structure* influence the motion from the first PNA. *This follows naturally from using the asymptotic method and should hold true for GR.*

5. Implementation. Comparison with a General-Relativistic Ephemeris

In order to use the equations of motion for the mass centers¹¹ so as to check the theory, we have to know the values of the parameters that enter these equations. These are the 0-order masses M_a of the bodies (here the major bodies of the solar system), the initial conditions of their motion, and the constant velocity \mathbf{V} of the global zero-order mass center of the solar system, with respect to the preferred frame E (and also the constant G).¹⁰ (Of course there is no parameter like \mathbf{V} in conventional theories.) These unknown parameters depend on the theory. They must be determined by optimizing the agreement between predictions and observations.¹⁰ Our computer code loops on the numerical solution of the translational equations of motion in order to optimize the parameters.¹⁵ This code has been tested by investigating in which measure one may reproduce (over one century) the predictions of the DE403 ephemeris,¹⁴ by using purely Newtonian equations of motion.¹⁵ It has also been applied to adjust over 60 centuries a less simplified model, in which the PN corrections in the Schwarzschild field of the Sun are also considered.¹⁶

In the version of the code that incorporates the equations of motion¹¹ derived from the present theory, a Lorentz transform allows to pass from the preferred reference frame to the frame bound with the zero-order global barycenter, and vice-versa. This transform, as well as the inverse transform, is determined by the adjustable vector \mathbf{V} . Thus, the adjustment process of the translational equations on observational data provides us eventually with the value of \mathbf{V} that minimizes the residual with the set of observations. Note that the “observational data” are currently taken from an *ephemeris based on GR*,¹⁴ specifically we take a set of heliocentric positions of the eight major planets, between 1956 and 2000. With these input data, themselves a fitting of observations by GR equations, the magnitude of the optimal vector \mathbf{V} is $|\mathbf{V}| \simeq 3$ km/s, which is significant. The difference between DE403 and our thus-adjusted equations of motion is small (cf. the residual advance in Mercury’s longitude of perihelion, with respect to Newton’s theory: 43”), but significant (Fig. 1. The self-rotation of all nine bodies is neglected). Our current project is to adjust the theory on a set of true *astronomical observations*.

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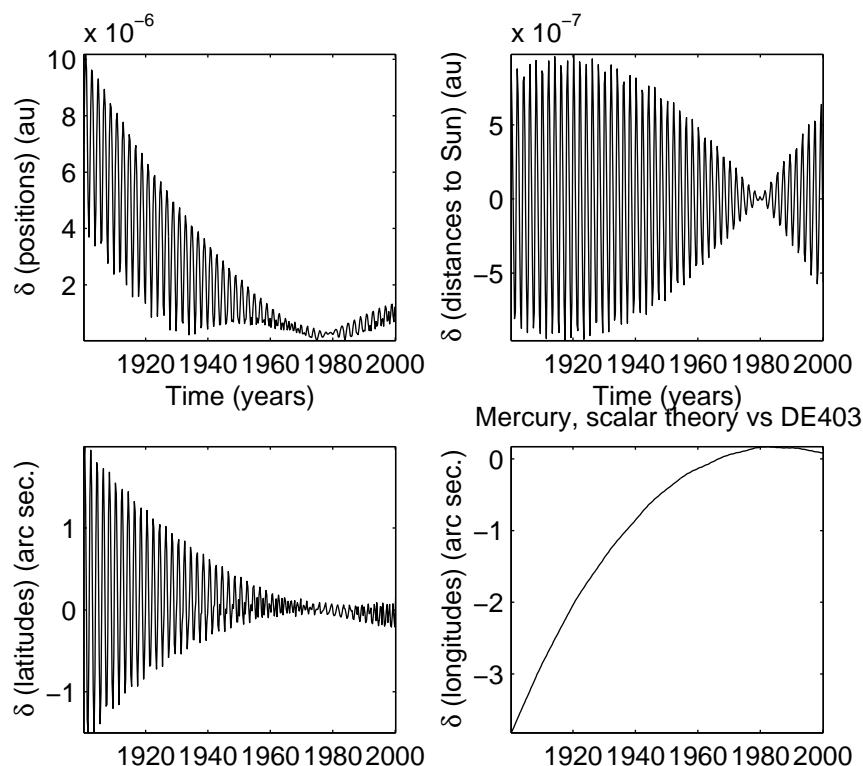


Fig. 1. Difference between the scalar theory and the DE403 ephemeris of the JPL.

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